

- Please attempt all questions and be concise in your answers.
- Time limit is **4 hours**. You may use any hand-written notes you took in class but no books. You may take several breaks as needed.
- Please write things and draw diagrams clearly as it will improve quality and grades.

Problem 1. (20 points) Theoretical limits on shear and fracture strengths are important in understanding the mechanical properties of ultra-strong material. Each sub-problem here is related to these theoretical limits. Problems 1(a-b) are related to each other, and problem 1(c) is a separate question. The material parameters of Au are listed below.

- Shear modulus: 27 GPa
- Lattice parameter: 4.080 Å
- Young's modulus: 78 GPa
- Surface energy: 1.2 J/m²
- Taylor coefficient: 0.5

(a) The upper limit on the shear strength of materials can be theoretically calculated without the concept of dislocations. In the presence of dislocations, however, the same shear strength can be attained via Taylor hardening. Estimate the dislocation density at which gold has the theoretical shear strength.

(b) Materials with high dislocation densities usually also have very high strengths due to the Taylor hardening. These materials also become extremely brittle because the nominally mobile dislocations cannot move due to the strong dislocation-dislocation interactions, which inhibit plastic flow. Based on your results in part (a), plot the shear stress vs. shear strain and write the fracture strength (same as the theoretical strength) and the fracture strain on the curve. Assume that no other flaws are present.

(c) (This problem is unrelated to parts (a-b)). A material cannot contain a crack larger than its own dimensions. Small-scale materials, such as nano-particles and nano-pillars, contain only tiny cracks (if at all), whose size is smaller than the sample diameter (nano-meter scale).

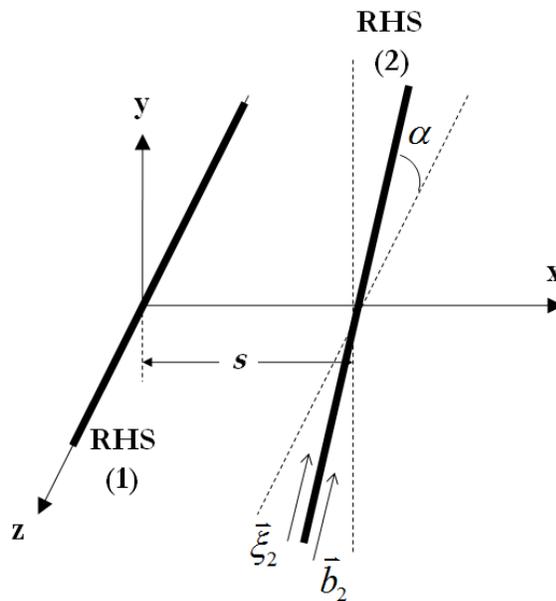
The Griffith criterion dictates that opening smaller cracks would require a higher stress, which implies that nano-particles should have very high fracture strengths. Eventually, if the particle diameter is below some critical value, fracture occurs via rupturing of the atomic bonds at one of the randomly distributed locations regardless of the existence of cracks. This occurs because the critical stress for crack growth is similar to the theoretical fracture strength due to the small dimension of particles and to the small crack sizes.

Estimate the maximum diameter of a spherical nano-particle where fracture occurs via the atomic bond severing as described above.

Problem 2 (20 points) Consider two pure RHS dislocations in the figure below. They lie in the planes parallel to the y - z plane but are separated at their closest point by a distance s . One dislocation lies along the z -axis while the other is tilted from a line parallel to z by the angle α . You need to decide the ranges of angles for the attractive, repulsive and no interactions between these two pure RHS dislocations in the x -direction. Assume that the dislocation (1) is fixed on the z -axis and that the magnitudes of Burgers vector of dislocation (1) and (2) are b_1 and b_2 , respectively.

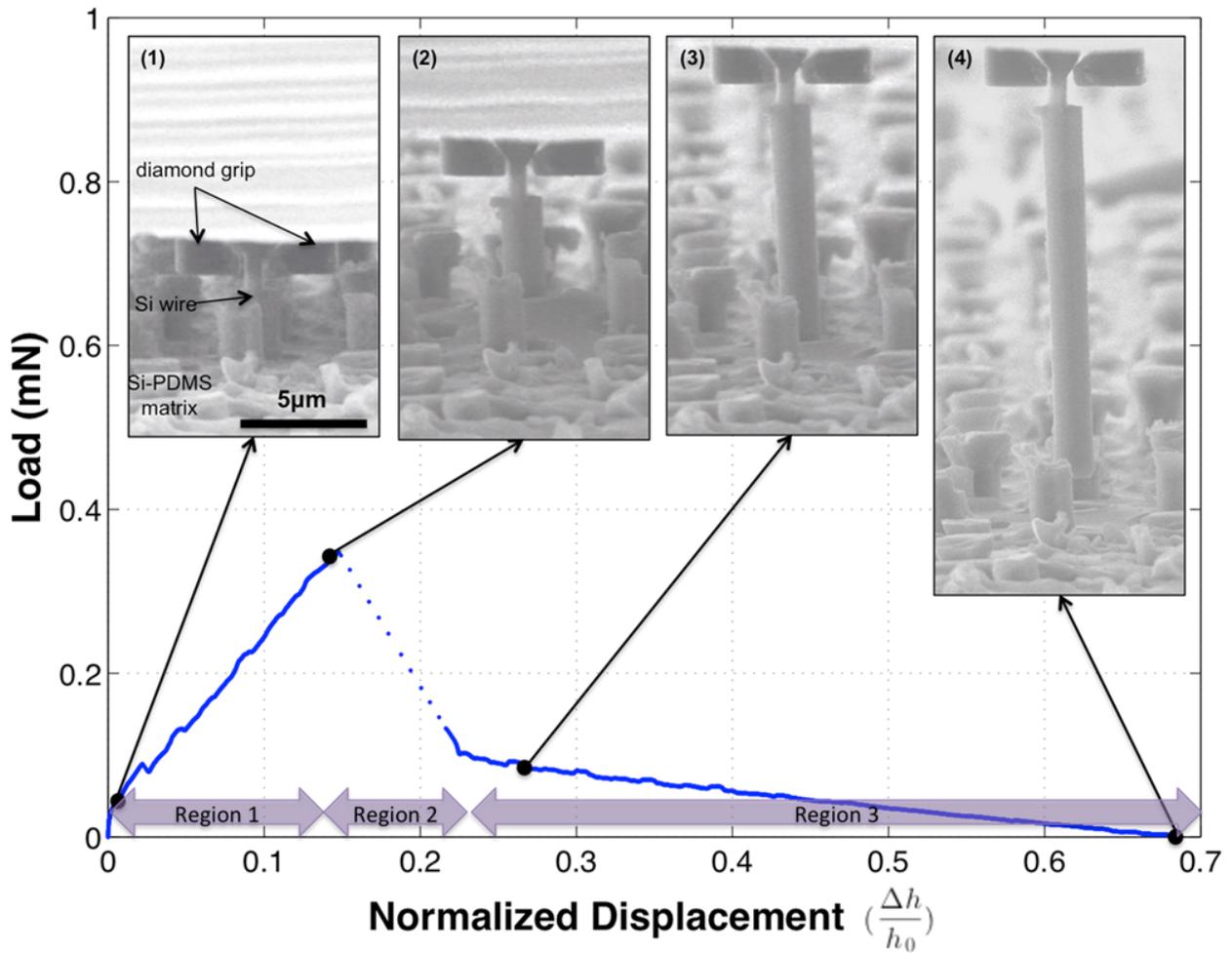
(Hint: to see whether these two dislocations are attractive, repulsive, or non-interactive in the x -direction, you need to only consider the x -component of the force on the dislocation (2).)

- Find the line sense vector and the Burgers vector of the dislocation (2) as a function of α .
- Express the x -component of the Peach-Koehler force (F_x) on dislocation (2) as a function of α .
- Obtain the ranges of angle, α , for the attractive, repulsive, and no force on dislocation (2). The overall range of angle, α , is from $0 \sim \pi$ because dislocation (2) becomes the same dislocation when it is rotated by π



Problem 3 (16 points). Calculate the relative density of: (a) a 2-dimensional honeycomb lattice composed of regular hexagons and (b) 3-dimensional octet lattice in terms of the length of the struts, l , and their thickness, t .

Problem 4 (27 points). The figure below shows an experiment where a Si microwire is being pulled out by applied tensile force from a polymer matrix.



- Convert this data into interfacial stress vs. normalized displacement assuming wire diameter of 7 microns and length of 100 microns.
- Determine the stress at the initial debonding event
- Calculate interfacial shear stress and the axial stress in the fiber using shear lag model.