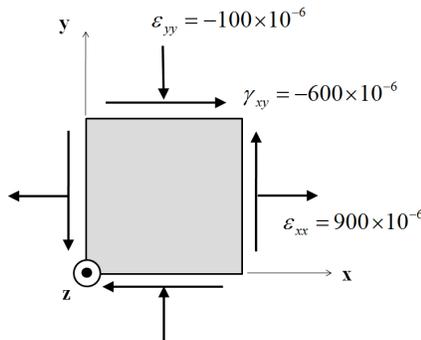


- Please attempt all questions and be concise in your answers.
- Time limit is **3 hours**. You may use any hand-written notes you took in class and the handouts but no books and no computers. You may take one to two breaks.
- Please write things and draw diagrams clearly as it will improve quality and grades.

**Problem 1 (16 Points).** Strains on a structural part were measured to be  $\epsilon_{xx} = 900 \times 10^{-6}$ ,  $\epsilon_{yy} = -100 \times 10^{-6}$  and  $\gamma_{xy} = -600 \times 10^{-6}$ . All other strain components are zero. The shear modulus,  $\mu$ , is 50 GPa and the Poisson's ratio,  $\nu$ , is 0.2. The material is elastically isotropic.

(a) Determine the principal strains,  $\epsilon_I$ ,  $\epsilon_{II}$ , and  $\epsilon_{III}$ , and the maximum shear strain,  $\gamma_{\max}$

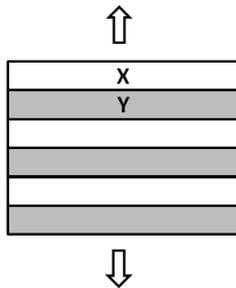
(b) Determine the principal stresses,  $\sigma_I$ ,  $\sigma_{II}$ , and  $\sigma_{III}$  and the maximum shear stress,  $\tau_{\max}$ .



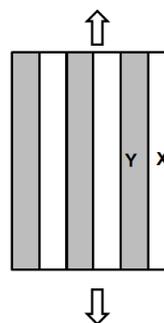
**Problem 2 (16 Points).** You are given six isotropic materials with different elastic properties (information provided below). Determine which one of these materials experiences the highest and the lowest stresses when the same amount of uniaxial elastic strain is applied. Assume that elastic properties do not change between 0 K and room temperature for material A.

• **Material A:**  $V_A(r) = 4\epsilon_A \left[ \left( \frac{\sigma_A}{r} \right)^{12} - \left( \frac{\sigma_A}{r} \right)^6 \right]$ ,  $\epsilon_A = 0.4 \text{ eV}$ ,  $\sigma_A = 3.8 \text{ \AA}$ .

• **Material B:**  $E_X = 80 \text{ GPa}$ ,  $E_Y = 140 \text{ GPa}$   
Volume fraction of X : 60 %  
Volume fraction of Y : 40 %

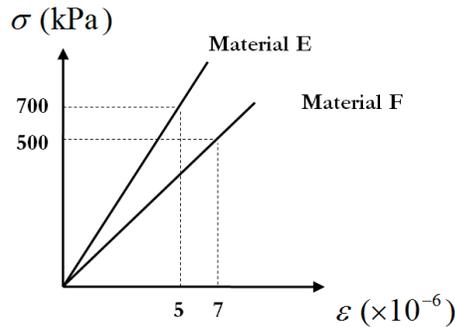


• **Material C:**  $E_X = 100 \text{ GPa}$ ,  $E_Y = 170 \text{ GPa}$   
Volume fraction of X : 80 %  
Volume fraction of Y : 20 %



• **Material D:** Lamé constant  $\lambda = 30$  GPa, Poisson's ratio  $\nu = 0.2$

• **Materials E and F:**



**Problem 3. (20 points).**

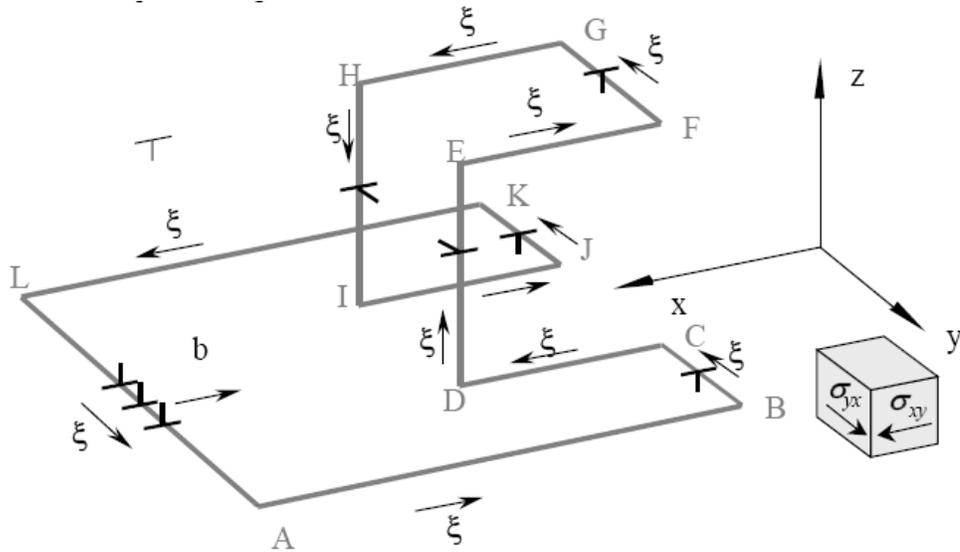
- (a) Write down a general relationship between the independent components of stress and strain for an elastically anisotropic crystal in terms of the compliance matrix  $[S_{ij}]$ .
- i) What is the meaning of the compliance constant,  $S_{15}$ ?
  - ii) Show that  $S_{15} = 0$  for crystals with cubic symmetry.
  - iii) Noting that the anisotropy ratio for most metals is less than 1, find the elastic moduli,  $E_{\langle 100 \rangle}$ ,  $E_{\langle 110 \rangle}$ , and  $E_{\langle 111 \rangle}$  and arrange them in increasing order.

**Problem 4. (16 Points).** Yielding in three different materials, A, B, and C, occurs under the same 3D multi-axial loading condition. The stress components were measured to be  $\sigma_{xx} = 20$  MPa,  $\sigma_{yy} = 20$  MPa and  $\tau_{xy} = 10$  MPa. This is the plane stress (2D) condition to perform stress transformation, but you need to solve this problem with the 3D stress state. The yield criterion of each material is:

- **Material A** : maximum principal stress
- **Material B** : maximum shear stress
- **Material C** : maximum distortion energy

Estimate the yield strengths ( $\sigma_{ys}$ ) of each material.

**Problem 5. (16 Points).** A non-planar dislocation loop is shown in the diagram below. All of the dislocation segments are parallel or perpendicular to the coordinate axes shown. The segment labeled L-A is a positive edge dislocation.



- (a) Determine the character of the segments C-D, D-E, E-F and F-G.
- (b) Assuming that all of the screw dislocation segments are locked and are not free to move, determine which of the dislocation segments will move in response to a positive shear stress,  $\sigma_{xy}$ . Indicate the direction of motion of those segments that will move in response to this shear stress.