Winter 2016

MS/ME/MedE 115 **Problem Set 1.** **J.R. Greer**

Problem 1.

Find the principal stress and the orientation of the principal stress axes for the following cases of plane stress:

(a) $s_x = 4{,}000$ MPa
$s_y = 0$
$s_{xy} = 8{,}000$ MPa

(b) $s_x = -12{,}000$ MPa
$s_y = 5{,}000$
$s_{xy} = -10{,}000$ MPa

Problem 2.

The stress state in a cube whose edges are parallel to the coordinate axes of an x-y-z system is constant and given by:

$$\begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

Consider the (010) and (111) planes and find for each:

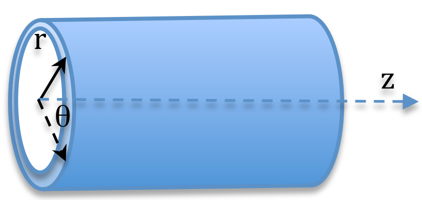
(a) The magnitude of the traction vector acting on the planes
(b) The magnitude of the normal traction acting on the planes
(c) The magnitude and direction of the shear traction acting on the planes.

Comment on the tractions on the (010) face of the plane if $s_{22} = 0$.

Problem 3.

Show that the principal stresses in a thin-walled closed and open-ended linear elastic cylinder in equilibrium, subject to an internal pressure p, are given by:

$\sigma_{zz} = pr/2t$ if the ends are closed, and
$\sigma_{zz} = 0$ if the ends are open
$\sigma_{\theta\theta} = pr/2t$, and
$\sigma_{rr} \approx 0$,



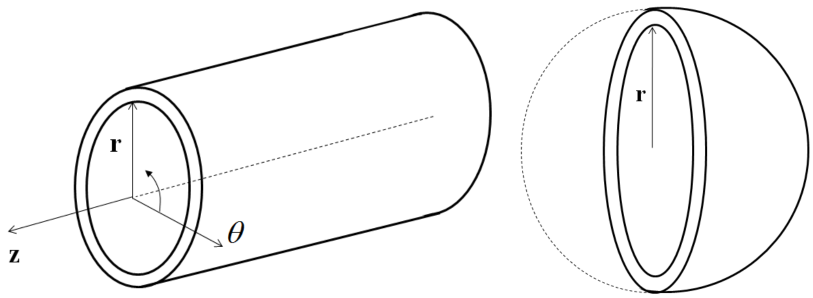
where r is the radius, L is the length , and t is the wall thickness (t<<r) of the vessel. State all assumptions.

Problem 4.

In the lecture notes, the principal stresses in a thin-walled closed-ended, linear elastic cylinder, subjected to an internal pressure *p*, at equilibrium were given by:

$$\sigma_{zz} = \frac{pr}{2t}, \quad \sigma_{\theta\theta} = \frac{pr}{t}, \quad \sigma_{rr} \sim 0 \text{ (for the outside of wall)}$$

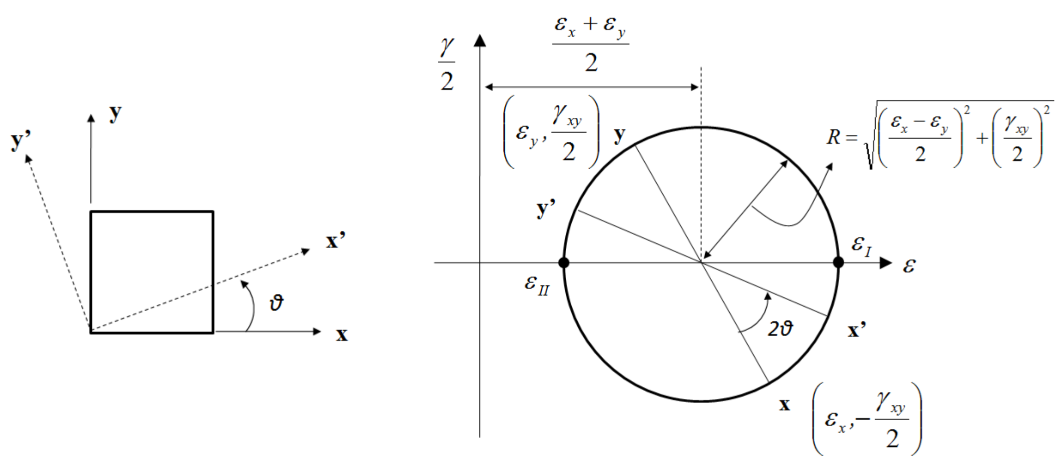
Drive similar relationships for the stresses in a thin-walled spherical pressure vessel of radius *r*. (Hint: Use $r-\theta-\phi$ coordinate system.)



Problem 5. We did not cover 2-dimensional 'strain' transformation in class, but the Mohr's circle can also be constructed for the 2D strain (rather than stress) state, which needs 3 independent components: ε_x, ε_y and γ_{xy}. The Mohr's circle for 2D strain transformation can be obtained simply by replacing σ_x, σ_y, τ_{xy} in the Mohr's circle for 2D stress transformation with ε_x, ε_y, $\frac{\gamma_{xy}}{2}$, respectively. In 2D strain state,

$$\text{The center of circle} = \left(\frac{\varepsilon_x+\varepsilon_y}{2}, 0\right)$$

$$\text{The radius of circle} = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2}$$



<Morh's circle of 2D strain transformation>

Suppose a solid sample is subjected to the following 2D strain state :

$\varepsilon_x = 340\times10^{-6}$, $\varepsilon_y = 110\times10^{-6}$, $\gamma_{xy} = 180\times10^{-6}$.

Calculate
(a) strains for an element rotated through an angle q = 30^o.
(b) principal strains
(c) maximum shear strain (engineering strain, not strain tensor)