Elastic Interactions of Dislocations

We have seen that forces are exerted on dislocations in crystals when they are subjected to external stresses. Similarly when the stress field of one dislocation acts on another, an interaction force (per unit length) is exerted on the second dislocation. And the stress field of the second dislocation creates forces on the first dislocation in the same way. Consider a straight dislocation (1) in the near proximity of a dislocation loop (2) as shown in the figure below. Using the Peach-Koehler formula we may say that the interaction force (per unit length) acting on dislocation (2) at the point $P$ is simply

$$F_{(2)} = -\vec{\xi}_{(2)} \times \left( \vec{b}_{(2)} \cdot \sigma_{ij}^{(1)} \right)$$

where $\vec{\xi}_{(2)}$ is the sense vector of dislocation (2) at the point $P$, $\vec{b}_{(2)}$ is the Burgers vector of dislocation (2) and $\sigma_{ij}^{(1)}$ is the stress field of dislocation (1) at the point $P$.

Dislocation-dislocation interaction

Examples

Parallel Screw Dislocations

Consider parallel RHS dislocations as shown in the diagram below. Dislocation (1) lies along the $z$ axis and dislocation (2) is parallel to it at the position $(r,\theta)$ or $(x,y)$. The sense vector for (2) may be chosen to be $\vec{\xi}_{(2)} = [0 \ 0 \ 1]$ so that $\vec{b}_{(2)} = b[0 \ 0 \ 1]$. The stress field of dislocation (1) is well known and can be expressed as
Screw-Screw interaction

\[
\sigma^{(1)}_{ij} = \begin{pmatrix}
0 & 0 & \sigma_{xz} \\
0 & 0 & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & 0
\end{pmatrix}
\]

where

\[
\sigma_{xz} = \sigma_{zx} = -\frac{\mu b_1}{2\pi} \frac{y}{x^2 + y^2}
\]

\[
\sigma_{yz} = \sigma_{zy} = \frac{\mu b_1}{2\pi} \frac{x}{x^2 + y^2}
\]

Using the Peach-Koehler formula we have

\[
\bar{F}_2(2) = -\bar{q}_2(2)x \left( \bar{b}_2(2) \cdot \sigma^{(1)}_{ij} \right)
\]

\[
\bar{F}_2(2) = -\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} b_2 \\ 0 \\ 0 \end{bmatrix} \begin{pmatrix} 0 & 0 & \sigma_{xz} \\ 0 & 0 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & 0 \end{pmatrix} = -b_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{pmatrix} \sigma_{zx} \\ \sigma_{zy} \\ 0 \end{pmatrix}
\]

\[
\bar{F}_2(2) = -b_2 \begin{bmatrix} x & y & z \\ \sigma_{zx} & \sigma_{zy} & 0 \end{bmatrix} = -b_2 \begin{bmatrix} -\sigma_{zy} & \sigma_{zx} & 0 \\ \sigma_{zx} & \sigma_{zy} & 0 \end{bmatrix}
\]

\[
\bar{F}_2(2) = \begin{bmatrix} \sigma_{zy} b_2 & -\sigma_{zx} b_2 & 0 \end{bmatrix}
\]

So the components of the interaction force are

\[
F_x = \sigma_{zx} b_2 = \frac{\mu b_1 b_2}{2\pi} \frac{x}{x^2 + y^2} = \frac{\mu b_1 b_2}{2\pi} \frac{\cos \theta}{r}
\]

\[
F_x = -\sigma_{zx} b_2 = \frac{\mu b_1 b_2}{2\pi} \frac{y}{x^2 + y^2} = \frac{\mu b_1 b_2}{2\pi} \frac{\sin \theta}{r}
\]
From this the radial interaction force may be computed as

\[ F_r = F_x \cos \theta + F_y \sin \theta = \frac{\mu b_1 b_2}{2\pi} \left( \cos^2 \theta + \sin^2 \theta \right) = \frac{\mu b_1 b_2}{2\pi r}. \]

We see that this result could have been found using the simple relations given earlier and some intuition. Since the shear stress about a RHS dislocation is simply \( \sigma_{z\theta} = \frac{\mu b_1}{2\pi r} \), the radial force on dislocation (2) can be computed using the basic relation, \( F = \tau b \), to be

\[ F_r = \sigma_{z\theta} b_2 = \frac{\mu b_1 b_2}{2\pi r} = \frac{\mu b_1 b_2}{2\pi r}, \]

which is, of course, the same result, as shown in the figure.

**Parallel Edge Dislocations**

We consider now edge dislocations of opposite signs lying on parallel slip planes as shown in the diagram below. We position dislocation (1), a positive edge dislocation, along the \( z \) axis and the oppositely signed dislocation (2) at the position \( (x, y) \). The forces acting on dislocation (2) due to the stress field of dislocation (1) can again be found using the Peach-Koehler formula. For this we choose the sense vector for dislocation (2) to be \( \mathbf{\hat{x}}_2 = [0 0 1] \) for which the Burgers vector is \( \mathbf{b}_2 = [1 0 0] \). The stress field of dislocation (1) is

\[ \sigma_{ij}^{(1)} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}. \]

Using the Peach-Koehler formula

\[ \bar{F}_2 = -\mathbf{\hat{x}}_2 \sigma_{ij} \mathbf{b}_2 \]

\[ \bar{F}_2 = -\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{b}_2 \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} -\sigma_{xx} \\ -\sigma_{xy} \end{bmatrix}. \]

\[ \bar{F}_2 = -b_2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -b_2 \begin{bmatrix} \sigma_{xy} \\ -\sigma_{xx} \end{bmatrix} \]

\[ \bar{F}_2 = \begin{bmatrix} -\sigma_{xy} b_2 \\ \sigma_{xx} b_2 \end{bmatrix}. \]
Edge-edge interaction. The inset shows how the glide and climb force can be obtained using the simple relations and intuition.

This result could have been obtained using the simple relations and some intuition. The diagram in the figure above shows that the glide force is

$$-F_g = -\sigma_{xy} b_2 = F_x,$$

and that the climb force is

$$F_c = \sigma_{xx} b_2 = F_y.$$

These results are consistent with the full Peach-Koehler analysis.

Using the known components of the stress field of a positive edge dislocation

$$\sigma_{xy} = \frac{\mu b_1}{2\pi(1-v)} \frac{x(x^2-y^2)}{(x^2+y^2)^2},$$

$$\sigma_{xx} = -\frac{\mu b_1}{2\pi(1-v)} \frac{y(3x^2+y^2)}{(x^2+y^2)^2},$$

the interaction forces on dislocation (2) are

$$F_x = -\sigma_{xy} b_2 = -\frac{\mu b_1 b_2}{2\pi(1-v)} \frac{x(x^2-y^2)}{(x^2+y^2)^2},$$

$$F_y = \sigma_{xx} b_2 = -\frac{\mu b_1 b_2}{2\pi(1-v)} \frac{y(3x^2+y^2)}{(x^2+y^2)^2}.$$
Interaction Forces in an Edge Dipole

The interaction forces between oppositely signed edge dislocations allows us the study the properties of edge dislocation dipoles and the circumstances under which they are in stable equilibrium with respect to glide or in stable equilibrium with respect to climb. The figure below shows the direction of glide and climb interaction forces on the second dislocation due to the stress field of the first, as calculated using the previous equations.

![Diagram showing interaction forces](image)

Properties of an edge dislocation dipole

The arrows denoted as $g$ and $c$ in each of the octants indicate the directions of glide and climb forces, respectively, on the second dislocation. The curved arrows indicate the path that the second dislocation might take relative to the first if it were able to both glide and climb in response to the interaction forces. As the curved arrows indicate, the ultimate fate of the second dislocation would be annihilation with the oppositely signed dislocation at the origin, if both glide and climb is allowed to occur. If the second dislocation lies anywhere along the dotted lines representing $|x| = |y|$, then the two dislocations are in stable equilibrium with respect to glide, not only because the glide force is zero at these places but also because the glide forces increase on either side of these positions to restore the dislocation to the $|x| = |y|$ line. Similarly, when the second dislocation lies along $y = 0$ the dislocations are in equilibrium with respect to climb for similar reasons. Of course the glide mobility is almost always much bigger than the climb mobility, so for most problems only glide need be considered. In that case the second dislocation will glide to the nearest $|x| = |y|$ line and remain there in stable equilibrium. Such an edge dipole is said to be in equilibrium with respect to glide. Only at high temperatures, where dislocations...
can climb by diffusional processes, could annihilation of the two dislocations occur.

**Critical Passing Stress for Edge Dislocations**

With the interaction forces we have calculated we can now determine the critical stress needed to cause two oppositely signed edge dislocations on parallel slip planes to move past each other. This is a fundamental problem in plasticity and strain hardening of crystals.

![Diagram of two edge dislocations](image)

**Bypassing of two edge dislocations**

We let dislocation (1) to be fixed at the origin and seek the critical applied shear stress, \( \tau_c \), that would be needed to cause the second dislocation, initially at \( x \gg h \), to move past the first dislocation. We can analyze this problem by first determining the applied shear stress, \( \tau \) that would be needed to hold the second dislocation at any point in its slip plane. The interaction force on dislocation (2) due to the stress field of dislocation (1) is

\[
F_{x}^{\text{int}} = -\sigma_{xy}^{(1)}b_2 = -\frac{\mu b_1 b_2}{2\pi(1-v)} \frac{x(x^2-y^2)}{(x^2+y^2)^2} = -\frac{\mu b_1 b_2}{2\pi(1-v)} \frac{x(x^2-h^2)}{(x^2+h^2)^2}.
\]

And the force due to the applied stress is

\[
F_{x}^{\text{applied}} = -\tau b_2.
\]

We find the applied stress needed to hold dislocation (2) in place by setting
\[ F_{x}^\text{int} + F_{x}^\text{applied} = 0 \]
\[ \tau = -\frac{\mu b_1}{2\pi(1-v)} \frac{x(x^2-h^2)}{(x^2+h^2)^2} - \tau b_2 = 0 \]

where \( \alpha = x/h \).

We can express this in non-dimensional form as

\[ \tau = \frac{2\pi(1-v)h\tau}{\mu b_1} = \frac{\alpha(\alpha^2-1)}{(\alpha^2+1)^2}, \]

which is plotted in the figure below.

Interaction stresses for passing edge dislocations

The interaction stresses in the above figure can be interpreted as follows:
Consider first that dislocation (2) starts off far to the right of the fixed dislocation (1), \( \alpha >> 1 \). Here the applied stress would have to be negative to prevent dislocation (2) from moving toward the oppositely signed dislocation (1). As dislocation (2) moves to the left and closer to dislocation (1), the attractive force grows larger and reaches a maximum at \( \alpha = 2.41 \) before declining to zero at \( \alpha = 1 \). This is a stable equilibrium position for the dipole. In order to push dislocation (2) still closer to dislocation (1) a positive shear stress needs to be applied. That required shear stress rises quickly and reaches \( \tau = 0.25 \) at
\( \alpha = 0.415 \) before declining to zero at \( \alpha = 0 \). The maximum in the shear stress represents the critical shear stress that would be needed to drive the two dislocations past each other. This critical passing stress, as discussed below, plays an important role in strain hardening.

As dislocation (2) moves to the left of dislocation (1) a negative shear stress is again needed to prevent the two dislocations from pushing away from each other to reach a new stable equilibrium position at \( \alpha = -1 \). A positive shear stress is required to move the oppositely signed dislocations away from each other, to \( \alpha << -1 \). Notice that the critical shear stress needed to pull the two dislocations apart is the same as the shear stress needed to make them pass in the first place.

It is easy to find the critical passing stress and the positions where that maximim interaction stress is found. These points (extrema) are characterized by the condition that

\[
\frac{\partial \tau}{\partial \alpha} = 0. 
\]

Taking

\[
\tau = \frac{\alpha(\alpha^2 - 1)}{(\alpha^2 + 1)^2} 
\]

from above we have

\[
\frac{\partial \tau}{\partial \alpha} = \frac{\left(\alpha^2 + 1\right)^2 \left(\alpha^2 - 1 + 2\alpha^2\right) - \alpha(\alpha^2 - 1)\alpha(\alpha^2 + 1)2\alpha}{\left(\alpha^2 + 1\right)^4} = 0 
\]

\[
\frac{\partial \tau}{\partial \alpha} = \frac{\left(\alpha^2 + 1\right)\left(3\alpha^2 - 1\right) - 4\alpha^2(\alpha^2 - 1)}{\left(\alpha^2 + 1\right)^3} = 0 
\]

so that

\[
3\alpha^4 - \alpha^2 + 3\alpha^2 - 1 - 4\alpha^4 + 4\alpha^2 = 0 \\
-\alpha^4 + 6\alpha^2 - 1 = 0 
\]

Letting \( \alpha^2 = w \) we have

\[
w^2 - 6w + 1 = 0
\]
which has the solutions
\[ w = \alpha^2 = \frac{6 \pm \sqrt{36 - 4}}{2} = 3 \pm \sqrt{8}, \]
so the roots are
\[ \alpha = \pm \sqrt{3 + \sqrt{8}} = \pm 2.41 \]
\[ \alpha = \pm \sqrt{3 - \sqrt{8}} = \pm 0.415. \]
Inserting \( \alpha = 0.415 \) into
\[ \tau = \frac{2\pi(1 - \nu)h\tau}{\mu b_1} = \frac{\alpha(\alpha^2 - 1)}{(\alpha^2 + 1)^2}, \]
gives the critical passing stress of
\[ \tau_c = \frac{2\pi(1 - \nu)h\tau_c}{\mu b_1} = \frac{0.415((0.415)^2 - 1)}{(0.415)^2 + 1)^2} = 0.25, \]
or
\[ \tau_c = \frac{\mu b_1}{8\pi(1 - \nu)h}. \]

**Taylor Hardening**

The basic result we have derived for the passing stress for two oppositely signed edge dislocations can be used to derive a basic formula for the strengthening effect of dislocations – called the Taylor hardening formula. Let us assume that the passing stress we have derived applies to the passing of oppositely signed edge dislocations in an array of the kind shown below. The passing stress when many dislocations are present differs from the result we have for just two dislocations, but only by a constant – the form is the same. For the simple array shown in the figure the density of dislocations, \( \rho \) (the number of dislocations per unit area), is of course related to the spacing between the slip planes, \( h \), as
\[ \rho = \frac{2}{(2h)^2} = \frac{1}{2h^2} \] or \( h = 1/\sqrt{2\rho} \). Inserting this into the formula for the passing stress gives
\[ \tau_c = \frac{\mu b_1}{8\pi(1 - \nu)h} = \frac{\sqrt{2}\mu b_1\sqrt{\rho}}{8\pi(1 - \nu)h} = \alpha mb_1\sqrt{\rho} \]
where $\alpha = \sqrt{2 / 8\pi (1 - \nu)} \approx 0.3$. The figure below shows that the shear strength of pure metals copper scale almost perfectly with the square root of the dislocation density, as described by the Taylor relation. This simple relation comes directly from the fact that the stress singularity of dislocations is of the form $\sigma_{ij} \sim r^{-1}$.

Oppositely signed edge dislocation in an array

Relationship between dislocation density and flow stress in copper showing the validity of the Taylor relation

**Other Dislocation Interactions Described by the Peach-Koehler Formula**

In the interactions described above the interacting dislocations were straight and parallel to each other. Of course, these are the simplest kinds of interactions.
Here we consider straight dislocations that are not parallel to each other and later we consider a case in which one of the dislocations is not straight.

First consider the two pure RHS dislocations in the figure below. They lie in planes parallel to the $yz$ plane but are separated at their closest point by a distance $s$. One dislocation lies along the $z$ axis while the other is tilted from a line parallel to $z$ by the angle $\alpha$. We know the elastic field of dislocation (1) so it is easy to calculate the forces on dislocation (2) using the Peach-Koehler formula. For this we define the sense vector for (2) as $\vec{\xi}_2 = [0 \sin \alpha \ -\cos \alpha]$ so that the Burgers vector is $\vec{b}_2 = b_2[0 \sin \alpha \ -\cos \alpha]$. We know that the stress field of dislocation (1) is

$$\sigma_{ij}^{(1)} = \begin{pmatrix} 0 & 0 & \sigma_{xz} \\ 0 & 0 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & 0 \end{pmatrix},$$

where

$$\sigma_{xz} = \sigma_{zx} = -\frac{\mu b_1}{2\pi} \frac{y}{x^2 + y^2},$$

$$\sigma_{yz} = \sigma_{zy} = \frac{\mu b_1}{2\pi} \frac{x}{x^2 + y^2}.$$
Using the Peach-Koehler formula, we have

\[
\vec{F}_2 = -\frac{\mu b_1}{2\pi} x \left[ \vec{\sigma}^{(1)}_{ij} \right]
\]

\[
\vec{F}_2 = -\left[ \begin{array}{c} 0 \\ \sin \alpha \\ -\cos \alpha \end{array} \right] \times b_2 \left[ \begin{array}{ccc} 0 & \sin \alpha & -\cos \alpha \\ -\cos \alpha & 0 & \sigma_{xz} \\ \sin \alpha & \sigma_{xy} & 0 \end{array} \right] \left( \begin{array}{c} 0 \\ 0 \\ \sigma_{xy} \end{array} \right)
\]

\[
\vec{F}_2 = -b_2 \left[ \begin{array}{ccc} x & 0 & y \\ \sin \alpha & -\cos \alpha & z \\ -\cos \alpha \sigma_{zx} & 0 & \sin \alpha \sigma_{zy} \end{array} \right] \left[ \begin{array}{ccc} 0 & \sin \alpha & -\cos \alpha \\ -\cos \alpha & 0 & \sigma_{xz} \\ \sin \alpha & \sigma_{xy} & 0 \end{array} \right]
\]

\[
\vec{F}_2 = -b_2 \left[ \begin{array}{ccc} x & y & z \\ \sin \alpha & -\cos \alpha & 0 \\ -\cos \alpha \sigma_{zx} & 0 & \sin \alpha \sigma_{zy} \end{array} \right] \left[ \begin{array}{ccc} 0 & \sin \alpha & -\cos \alpha \\ -\cos \alpha & 0 & \sigma_{xz} \\ \sin \alpha & \sigma_{xy} & 0 \end{array} \right]
\]

It is interesting to focus on the \( x \) component of the interaction force, \( F_x \).

\[
F_x = b_2 \left( \sin^2 \alpha - \cos^2 \alpha \right) \sigma_{zy}.
\]

Notice that this produces the following results:

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( F_x )</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0 )</td>
<td>( \sigma_{zy} b_2 )</td>
<td>repulsive</td>
</tr>
<tr>
<td>( \alpha = \pi / 4 )</td>
<td>( 0 )</td>
<td>no interaction</td>
</tr>
<tr>
<td>( \alpha = \pi / 2 )</td>
<td>( -\sigma_{zy} b_2 )</td>
<td>attractive</td>
</tr>
</tbody>
</table>

Obviously, as we have seen before, like-signed screws repel each other when they are parallel to each other (\( \alpha = 0 \)). But we now find that like-signed screws attract each other when they are orthogonal (\( \alpha = \pi / 2 \)).

For the orthogonal case, \( \alpha = \pi / 2 \), the distribution of interaction forces is not uniform. The stress field of dislocation (1) is

\[
\sigma_{zy} = \frac{\mu b_1}{2\pi} \frac{x}{x^2 + y^2}
\]
so the $x$ component of the interaction forces is expressed as

$$F_x = -\sigma_{xy} b_2 = -\frac{\mu b_1 b_2}{2\pi} \frac{x}{x^2 + y^2}$$

or, for $x = s$, is

$$F_x = -\frac{\mu b_1 b_2}{2\pi} \frac{s}{s^2 + y^2},$$

as shown in the sketch below.

Interaction forces between orthogonal screw dislocations

The Peach-Koehler formula may also be used to compute the forces on curved dislocations of the kind shown in the diagram below. Here an edge dislocation loop is near a positive edge dislocation.
The sense vector and Burgers vector of the loop can be expressed as

\[ \hat{\mathbf{z}} = \begin{bmatrix} 0 & \sin \alpha & -\cos \alpha \end{bmatrix} \] and \[ \mathbf{b}_2 = b_2 \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \]

and the stress field of dislocation (1) is

\[ \sigma_{ij}^{(1)} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}. \]

So the interaction forces can be found using the Peach-Koehler formula

\[ \bar{F}_2 = -\hat{\mathbf{z}}(2) \times (\mathbf{b}_2 \cdot \sigma_{ij}^{(1)}) \]

\[ \bar{F}_2 = -\begin{pmatrix} 0 & \sin \alpha & -\cos \alpha \end{pmatrix} \begin{bmatrix} b_2 \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \end{bmatrix} = -b_2 \begin{bmatrix} 0 & \sin \alpha \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{xy} \end{bmatrix} \]

\[ \bar{F}_2 = -b_2 \begin{pmatrix} x & y & z \\ \sigma_{xx} & \sigma_{xy} & 0 \end{pmatrix} = -b_2 \begin{pmatrix} \sigma_{xy} \cos \alpha & -\sigma_{xx} \cos \alpha & -\sigma_{xx} \sin \alpha \end{pmatrix} \]

\[ \bar{F}_2 = \begin{pmatrix} -\cos \alpha \sigma_{xy} b_2 & \cos \alpha \sigma_{xx} b_2 & \sin \alpha \sigma_{xx} b_2 \end{pmatrix} \]

We can use this result to describe the attractive interaction when the dislocation loop is just above a positive edge dislocation.

Dislocation loop (vacancy loop) above a positive edge dislocation

For a loop in this location the relevant stresses from dislocation (1) are
\[ \sigma_{yy} = 0 \]
\[ \sigma_{xx} = -\frac{\mu b_2}{2\pi(1-\nu)}\frac{1}{y} = -\frac{\mu b_1}{2\pi(1-\nu)}\frac{1}{(s - R\cos\alpha)} . \]

From the result above, the \( y \) and \( z \) components of the interaction force (per unit length) are

\[ F_y = \cos\alpha \sigma_{xx} b_2 = -\frac{\mu b_2}{2\pi(1-\nu)}\frac{\cos\alpha}{(s - R\cos\alpha)} \]

\[ F_z = \sin\alpha \sigma_{xx} b_2 = -\frac{\mu b_2 b_1}{2\pi(1-\nu)}\frac{\sin\alpha}{(s - R\cos\alpha)} \]

These forces tend to cause the loop to expand, non-uniformly, as shown by the diagram above. It is obvious from these results that since \( F_z \) is an odd function of \( \alpha \) the net force on the entire loop in the \( z \) direction is zero. But the net force on the loop in the \( y \) direction is not zero; it is attractive because the attraction forces for the part of the loop nearest the edge dislocation are greater than the repulsive forces on the part of the loop far from the edge dislocation. The total force on the entire circular loop can be computed as follows

\[ F_{y\text{total}} = \frac{2\pi}{\nu} \int_0^{\pi} F_y R\,d\alpha = -\frac{\pi}{\nu} \frac{\mu b_2 b_1}{2\pi(1-\nu)}\frac{\cos\alpha}{(s - R\cos\alpha)} R\,d\alpha \]

\[ F_{y\text{total}} = -\frac{\mu b_2 b_1}{\nu(1-\nu)} \int_0^{\pi} \frac{\cos\alpha\,d\alpha}{((s/R) - \cos\alpha)} \]

\[ F_{y\text{total}} = \frac{\mu b_2 b_1}{(1-\nu)} \left( 1 - \frac{\alpha}{\sqrt{\alpha^2 - 1}} \right) \]

If the dislocations were able to climb, then the loop would extend toward the straight edge and the edge would bulge up toward the loop so that eventually the loop would annihilate in the dislocation. After a very long period of time we would have again a straight edge dislocation at a higher elevation, due to the absorption of the vacancy loop.