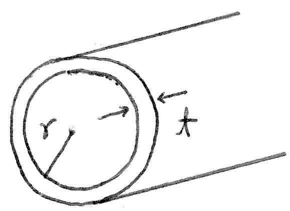
Thin walled pressure vessel.

purpose : to evaluate a plane stress condition of a thin-walled pressure vessel.

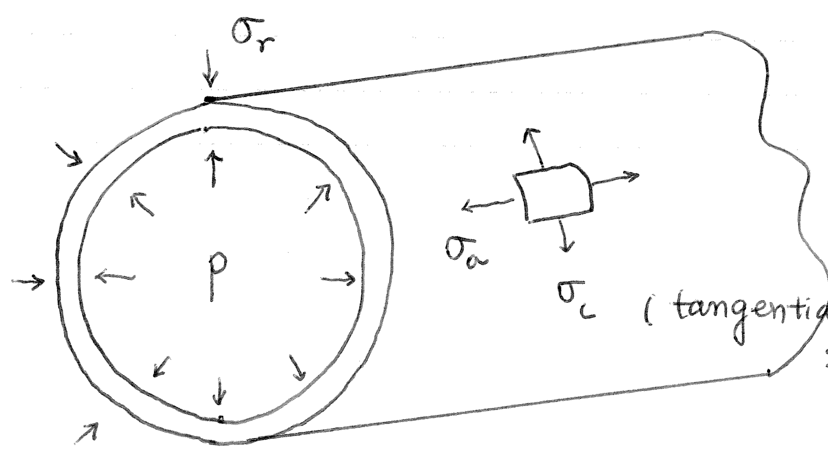
pressure vessel : It is a closed structure that contains liquid or gases under pressure.
e.g.) Boiler, air tank, pressurized pipe, pop-can

How thin is thin ?



in general, $\frac{t}{r} < \frac{1}{10}$.

Cylindrical pressure vessel.



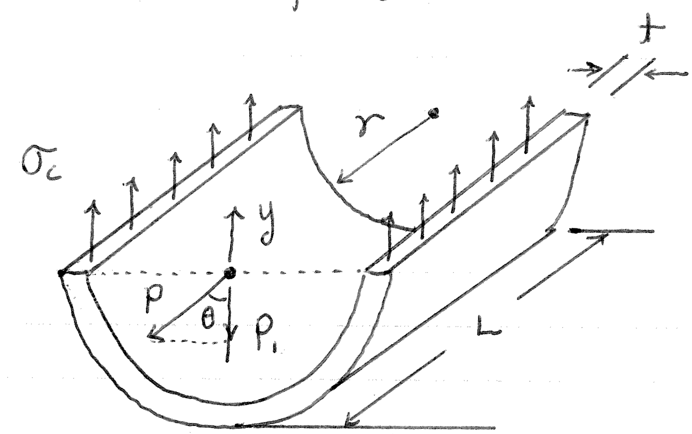
Axial direction (z).

(tangential direction to the surface of the cylinder.

σ_c : Tangential (Circumferential, hoop) stress $= \sigma_{\theta\theta}$.
σ_a : Axial (longitudinal) stress $= \sigma_{zz}$
σ_r : Radial stress $= \sigma_{rr}$

σ_c, σ_a, and σ_r can be calculated from equilibrium using appropriate free-body diagram.

- Calculation of σ_c



(Tangential force due to the internal pressure)
= (Resisting force by the vessel).

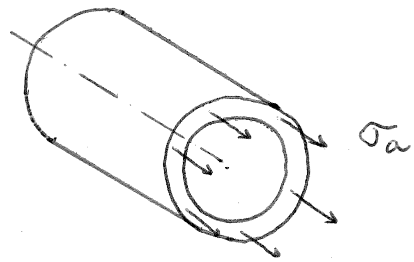
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} P_i r L \, d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} p r L \cos\theta \, d\theta = 2 p r L$$

$$= \sigma_c (2Lt).$$

$$\therefore \boxed{\sigma_c = \frac{Pr}{t}} = \sigma_{\theta\theta}$$

Note : σ_c is uniformly distributed over the thickness of the wall provided the wall is very thin.

- Calculation of σ_a (with the end caps).



From the force equilibrium

$$p(\pi r^2) = \sigma_a (2\pi r t).$$

$$\boxed{\sigma_a = \frac{pr}{2t}.} = \sigma_{zz}$$

- The radial stress $\left(\begin{array}{l} \sigma_r = -p \text{ (Acting on the inner surf).} \\ \sigma_r = 0 \text{ (Acting on the outer surf).} \end{array} \right.$

Thus, the stress state on the inner and outer surfaces.

Stress	Inner Surface	Outer surface
σ_1	$\sigma_c = \frac{pr}{t}$	$\sigma_c = \frac{pr}{t}$
σ_2	$\sigma_a = \frac{pr}{2t}$	$\sigma_a = \frac{pr}{2t}$
σ_3	$\sigma_r = -p$	$\sigma_r = 0$

biaxial stress state