and can be applied in some cases. We shall mainly consider the effects
and consequences of such polymers. Although real polymers have the physical
characteristics of solids, they are not firm structures. For example, some polymers are flexible and take a random coil structure.

3.8 Rodlike Polymers

B. Solutions of Rigid

Molecular Theory of Polymer Liquids
The coefficient $\mathcal{N}$ is called the rotational friction constant.

$$\mathcal{N} = \frac{\eta}{l}$$

where $\eta$ is the viscosity, and $l$ is a characteristic length, which is parallel to the axis of rotation.

The rotational friction force $F$ is given by:

$$F = -\mathcal{N} \frac{d\Phi}{dt}$$

where $\Phi$ is the angular displacement.

The rotational friction torque is:

$$T = I \frac{d\omega}{dt}$$

where $I$ is the moment of inertia, and $\omega$ is the angular velocity.

In summary, the rotational friction coefficient is a measure of the resistance to rotation, and it is important in understanding the behavior of rotating systems.
\[ A + \frac{1}{A} = A \]

and

\[ A^* + \frac{1}{A^*} = \gamma (\mathbf{e}) \]

Huygens' principle is the translation function concept. From eqns (8.14) and (8.15), we have the smoluchowski equation for

\[ \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = \frac{\partial}{\partial t} \left( \rho \mathbf{v} \right) + \nabla \cdot \left( \rho \mathbf{v} \nabla \mathbf{v} \right) - \nabla \cdot \left( \rho \mathbf{v} \nabla \rho \right) \]

\[ = \left( \mathbf{v} \cdot \nabla \right) \mathbf{v} + \nabla \left( \frac{\rho}{2} \mathbf{v} \mathbf{v} \right) \]

For given \( \mathbf{v} \) changes with the velocity \( \mathbf{v} \) and the equation for the

\[ \mathbf{n} \cdot \mathbf{v} \mathbf{n} + \left( \mathbf{n} + \frac{\partial}{\partial t} \right) \mathbf{n} \mathbf{n} + \frac{\partial}{\partial t} \left( \mathbf{n} \mathbf{n} \mathbf{n} \right) \]

velocity \( \mathbf{v} \) is given by

\[ \nabla \mathbf{v} = \gamma \mathbf{v} \mathbf{v} \mathbf{v} \]

The equation of motion of the hydrodynamics is considered next. Suppose a little of thefluidmotion into the small. 3. Translation motion

\[ \text{TRANSLATIONAL DIFFUSION} \]

This consequence of the hydrodynamics corresponds to eqn (8.15)

\[ \mathbf{n} \cdot \mathbf{n} + \left( \mathbf{n} + \frac{\partial}{\partial t} \right) \mathbf{n} \mathbf{n} = \frac{\partial}{\partial t} \left( \mathbf{n} \mathbf{n} \right) \]

velocity \( \mathbf{v} \) is given by an external potential \( \phi \) is given as

\[ \mathbf{v} = \nabla \phi \]

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velocity \( \mathbf{v} \) is given by an external potential \( \phi \) is given as

\[ \mathbf{v} = \nabla \phi \]
and an example of the application of the Brownian dynamics is shown in Fig. 23.

However, the real test of the model for the Brownian motion is its success in reproducing the observed facts. In the case of a rod, the rod-motion equation is

\[ \dot{\mathbf{r}} + \mathbf{f}(t) = \frac{m}{\gamma} \mathbf{a}(t) \]

where \( \mathbf{f}(t) \) is the force on the rod and \( \mathbf{a}(t) \) is the acceleration of the rod.

We can now write down the Smoluchowski equation which includes both the rotational and translational motion.

**Smoluchowski equation including both translational and rotational motion**

This is the result of hydrodynamic calculation.

The conservation of \( D \) and \( D^T \) characteristic the diffusion parallel and perpendicular to the rod.

The calculation of \( \frac{\tau}{D^T} \) and \( \frac{\tau_1}{D^T} \) is given in Appendix 8.1. The result is based on the Kirkwood theory as given in Eq. (82).

The calculation of \( \frac{T_1}{D} \) and \( \frac{T}{D^T} \) is given in Eq. (82).

We have

\[ \frac{1}{\gamma} - \lambda = \gamma \] and \( n(n - A) = n \]

Substituting

\[ \lambda \frac{T}{D^T} + \frac{1}{\gamma} \frac{T}{D} = f \]

Thus, it follows that the distribution function for the rod in the configuration \( (r, n, A) \) are not equal to each other.
\[ (n-n)\rho (\mathbf{r} - \mathbf{r}') \rho = (i^t \cdot \nabla)^2 \psi (x) \rho \\
\text{with the initial condition} \\
\frac{d\rho}{d\tau} = \frac{\nabla \cdot ((n-n)^t \mathbf{Q} + n (n-n)^t \mathbf{Q}) \rho}{\rho} + \frac{\nabla \cdot \mathbf{Q} \rho}{\rho} = \frac{\partial \rho}{\partial \tau} \]

Consider the mean square displacement of the center of mass:

\[ \langle (\langle (1 \cdot \mathbf{r} \rangle)^2 \rangle \rangle = (i^t)^t \psi (x) \rho \]

where \( \psi (x) \) is the Legendre polynomial of \( n \). In order

\[ (i^t \cdot u^t \mathbf{Q} - \mathbf{Q}) \cdot \mathbf{r} = ( (\langle (1 \cdot (1 \cdot n) \rangle \rho \rangle \rangle)^t \psi (x) \rho \]

in general, we can show

\[ (i^t \cdot u^t \mathbf{Q} - \mathbf{Q}) \cdot \mathbf{r} = ( (\langle (1 \cdot (1 \cdot n) \rangle \rho \rangle \rangle)^t \psi (x) \rho \]

in the same way, one can show

\[ (i^t \cdot u^t \mathbf{Q} - \mathbf{Q}) \cdot \mathbf{r} = ( (\langle (1 \cdot (1 \cdot n) \rangle \rho \rangle \rangle)^t \psi (x) \rho \]

For \( i > 0 \), see (8a) for general form, which gives a clear physical

\[ ((0 \cdot (1 \cdot n) \rangle \rho \rangle \rangle)^t \psi (x) \rho \]

From (8a), it follows that

\[ ((i^t \cdot u^t \mathbf{Q} - \mathbf{Q}) \cdot \mathbf{r} = ((0 \cdot (1 \cdot n) \rangle \rho \rangle \rangle)^t \psi (x) \rho \]

The rotational correlation time \( \tau \) is the same

\[ (i^t \cdot u^t \mathbf{Q} - \mathbf{Q}) \cdot \mathbf{r} = ((0 \cdot (1 \cdot n) \rangle \rho \rangle \rangle)^t \psi (x) \rho \]

Since (8a) is equal to \( (0 \cdot (1 \cdot n) \rangle \rho \rangle \rangle)^t \psi (x) \rho \]

\[ (i^t \cdot u^t \mathbf{Q} - \mathbf{Q}) \cdot \mathbf{r} = ((0 \cdot (1 \cdot n) \rangle \rho \rangle \rangle)^t \psi (x) \rho \]

Hence eqn (8a) is written as

\[ (i^t \cdot u^t \mathbf{Q} - \mathbf{Q}) \cdot \mathbf{r} = ((0 \cdot (1 \cdot n) \rangle \rho \rangle \rangle)^t \psi (x) \rho \]

Applying this twice, we have

\[ \langle (\langle (1 \cdot \mathbf{r} \rangle)^2 \rangle \rangle = (i^t)^t \psi (x) \rho \]

The direction of the axis \( \mathbf{Q} \) is fixed, hence

\[ \langle (\langle (1 \cdot \mathbf{r} \rangle)^2 \rangle \rangle = (i^t)^t \psi (x) \rho \]

By a straightforward calculation

\[ (i^t)^t \psi (x) \rho \]

L avenue (8a) for the right-hand side, we get

\[ (i^t)^t \psi (x) \rho \]

This probability is the average function of the diffusion equation

\[ (i^t)^t \psi (x) \rho \]

This is obtained by setting \( \mathbf{Q} \) in the direction of \( \mathbf{r} \), and

\[ \langle (\langle (1 \cdot \mathbf{r} \rangle)^2 \rangle \rangle = (i^t)^t \psi (x) \rho \]

Having obtained the Smoluchowski equation, we now study the case

\[ \langle (\langle (1 \cdot \mathbf{r} \rangle)^2 \rangle \rangle = (i^t)^t \psi (x) \rho \]

8.4 Brownian motion in the equilibrium state

**Brownian motion in the equilibrium state**
corresponding to the solutions of the expression of (3.2). Let $\phi$ and $\psi$ be the eigenfunctions of the formal integration of this equation by corresponding the eigenfunction

\[ \exp(-\int_C^x) = (1, x) \phi \]

(6.62)

Pekeln diffusion with the diffusion constant $C$, so that

In this case, it is initially given by the initial condition

\[ \frac{\partial}{\partial x} \phi(x, y) = 0 \]

(1.6)

with

\[ \phi(0, y) = \psi(y) \]

(1.8)

From equation (6.5.8) and (6.5.9), we get

\[ \phi(\eta, \ne) = \phi(0, \ne) \exp(-\int_0^\eta \phi(x, y) dx) \]

(6.5.8)

Then

\[ \phi(\eta, \ne) \]

(1.8)

is given by

\[ \phi(\eta, \ne) = \phi(0, \ne) \exp(-\int_0^{\eta/n} \phi(x, y) dx) \]

(6.5.8)

Thus is expressed by

\[ \phi(\eta, \ne) = \phi(0, \ne) \exp(-\int_0^{\eta/n} \phi(x, y) dx) \]

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Then is obtained from

\[ \phi(\eta, \ne) = \phi(0, \ne) \exp(-\int_0^{\eta/n} \phi(x, y) dx) \]

(6.5.8)

Since

\[ \phi(\eta, \ne) = \phi(0, \ne) \exp(-\int_0^{\eta/n} \phi(x, y) dx) \]

(6.5.8)
9

SEMICILUTE SOLUTIONS OF RIGID RODLIKE POLYMERS

9.1 Semidilute and concentrated solutions of rodlike polymers

In Chapter 8, we discussed the dynamics of a single rodlike polymer. Let us now consider the interaction between the polymers at finite concentration.

Solutions of slender rodlike polymers of length $L$ and diameter $b$ may be classified into four concentration regimes (Fig. 9.1). Let $\rho$ be the weight of polymers in unit volume of the solution, then the number of polymers per volume is given by

$$n = \frac{\rho}{M} N_A$$

(9.1)

where $M$ is the molecular weight.

(i) Dilute solution (Fig. 9.1a). A dilute solution is defined as one having a sufficiently low concentration that the average distance between the polymers $n^{-1/3}$ is much larger than $L$, i.e.,

$$n \ll n_1 \sim 1/L^3$$

(9.2)

In such a solution each polymer can rotate freely without interference by other polymers. The effect of the interaction can be expressed by a power series expansion with respect to $n$ as in the case of flexible polymers.

(ii) Semidilute solution (Fig. 9.1b). If $n \gg n_1$ the rotation of each polymer is severely restricted by other polymers, so that the dynamics of the polymers will be entirely different from that in dilute solution. However, the static properties will not be affected seriously until the concentration reaches another characteristic concentration $n_2$. This is easily seen if one considers that the polymers are mathematical lines with no thickness. The equilibrium distribution of such polymers is entirely independent of each other at all concentrations. Thus the effect of the interaction becomes important in static properties only for polymers with finite diameter. Indeed the excluded volume of rigid rods is shown to be of the order of $bL^2$ (see Fig. 9.2)), so that the static properties are unaffected if $n \ll n_2$ is small. We call the concentration regime

$$n_1 \ll n \ll n_2 = 1/bL^2$$

(9.3)