Problem Set #1

Due Friday April 8, 2016 at 5 pm

MS133: Kinetic Processes in Materials

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1. A semi-infinite bar of α phase of composition $c_{\alpha\infty}$ is placed in contact with a semi-infinite bar of ϵ phase of composition $c_{\epsilon\infty}$. As these binary alloys diffuse together a third phase, β, is formed. The composition of α in contact with β is $c_{\alpha\beta}$, the composition of β in contact with α is $c_{\beta\alpha}$, the composition of β in contact with ϵ is $c_{\beta\epsilon}$ and the composition of ϵ in contact with β is $c_{\epsilon\beta}$.

   (a) In the region occupied by each phase, assume a solution of the form:

   $$c_i(z,t) = A_i + B_i \operatorname{erf}\left(\frac{z}{\sqrt{4D_i t}}\right) \tag{1.31}$$

   and solve for the coefficients B_α, B_β, and B_ϵ.

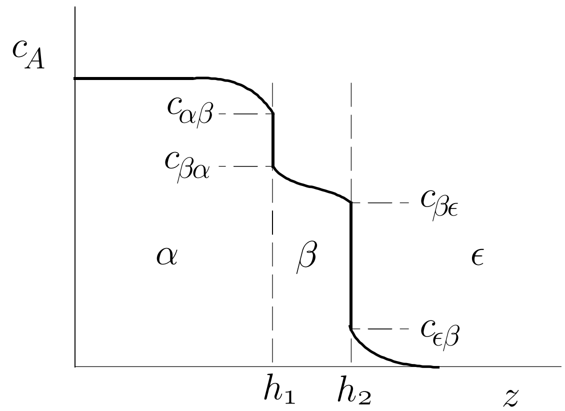

Figure 1.14: Schematic of diffusion couple concentration profile in a system where there is compound formation

(b) Write down the mass conservation equation for both interfaces.

(c) Given that the position of the interfaces, h_1 and h_2, are given by:

$$h_1 = \gamma_1\sqrt{4D_\beta t} \tag{1.32}$$

$$h_2 = \gamma_2\sqrt{4D_\epsilon t} \tag{1.33}$$

describe how you would find the width of the β region as a function of time.

(d) Assume that $D_\alpha = D_\epsilon = 0$, so that there is no diffusion in the α and ϵ regions, and assume that the composition in the β phase is linear between the limits $c_{\beta\alpha}$ and $c_{\beta\epsilon}$. Find the width of the β region as a function of time.

2. We place a slab of iron of thickness d and initial carbon concentration c_0 in a decarborizing oven which holds the surface carbon concentration at zero.

   (a) Use the separation of variables technique to find the concentration of carbon as a function of time and position within the slab. Consider the slab large enough such that we can ignore end effects.

   (b) Often mechanical properties are determined by the average composition of carbon. Find the average composition in the slab.

   (c) Find the ratio of successive terms for the average composition. How does this compare with that of the local concentration? For what average composition does using only the first term give an error of 1% or less.

   (d) Considering only the first term, what is the relaxation time for the removal of carbon from this slab?

3. Matt L. Science is a young engineer at the BMC Nuclear Power Plant in Pasadena, CA. Matt's supervisor is concerned that traces of radioactive sodium in the cooling water will diffuse through the stainless steel pipes and present a health hazard to workers. He has assigned Matt the job of determining the flux of sodium per unit length of pipe (J/L). The analysis lab has told them that there is a concentration of c_o on the outer surface of the pipe, and that there is a concentration c_i on the inner surface. The pipes have outer radius r_o and inner radius r_i. The diffusion coefficient of sodium in stainless steel is D and may be assumed to be independent of concentration.

   (a) At steady state, what is the flux per unit length of pipe?

   Matt is slightly naive and treats the problem as diffusion through a flat plate of thickness $r_o - r_i$ and area per unit length the same as that of the pipe. He assumes that this will be a reasonable approximation of the truth.

   (b) Will the flux which Matt predicts be higher or lower that the actual value? By how much?

   (c) On the same plot, sketch concentration vs. radius (for the cylindrical case) and concentration vs. thickness (for the planar case). Is Matt's approximation better for thin or thick wall pipes?

4. In a 1995 Physical Review article Fuchs et. al (Vol 51, page 16817-21) used secondary ion mass spectrometry to measure the concentration profile of Ge isotope heterostructures. By performing these measurements after various annealing treatments they were able to determine the diffusion coefficient. We imagine that we are there when they began this work and are asked to help determine the sample configuration and annealing times and temperatures by our knowledge of diffusion phenomena and from our best estimates of the diffusivity.

   The sample consists of a thin layer of ^{74}Ge between two thick layers of ^{70}Ge.

   (a) Assuming that initially the ^{74}Ge is in a very thin region surrounded by thick ^{70}Ge, write down the solution for the concentration of ^{74}Ge as a function of time and position. Assume that there is an amount m of ^{74}Ge per unit area.

   (b) Find the expression for the width that the distribution of ^{74}Ge will spread during an anneal of duration t_A. (Pick your own definition of "width," but be sure to tell me how you define it.)

   (c) Find an analytical expression for the temperature for which the width will reach a particular value during the anneal. Define carefully the parameters in your answer.

   (d) Using information in the notes or handouts for this class, estimate the diffusion coefficient of Ge as a function of temperature. Cite the source of any information used. What is this diffusion coefficient called?

   (e) Find the temperature that will result in a width of 0.1 μm during a 1 hour anneal. What is the value of the diffusivity at this temperature?